Physics 200A **Mechanics I** **Fall 2016**

Problem Set V: Due Wednesday, November 30, 2016

FW=Fetter and Walecka

1.) For a scalar field $\phi(\underline{x},t)$ with Lagrangian density $\mathcal{L} = \mathcal{L}(\phi, \partial_t\phi, \underline{\nabla}\phi)$,

a.) Derive the general Lagrangian equations of motion.

b.) Define a Hamiltonian density and derive the Hamiltonian equations of motion.

c.) For $\delta\mathcal{L}/\delta\underline{\nabla}\phi = -\gamma P_0 \underline{\nabla}\phi$, $\delta\mathcal{L}/\delta\partial_t\phi = \rho_0 \partial_t\phi$ $\delta\mathcal{L}/\delta\phi = 0$, derive the EOM. What physical system might this correspond to? What is the physical meaning of ϕ ?

d.) Derive the energy theorem for the system in c.).

2.) Consider a string of length L and mass-per-length μ which is, as usual, clamped at both ends. Assume the tension is T.

Express the Hamilton in terms of the Fourier coefficients, thereby converting the problem to one of particle dynamics. (Hint: Expand the displacement in terms of the *spatial* eigenfunctions.) Derive the Hamiltonian EOMs. What do these equations correspond to in Quantum Mechanics?

3.) a.) Generalize the derivation of the nonlinear wave equation for a string to that for a 2D membrane (i.e. drum head), with clamped boundary. Show that you recover the wave equation in the linear limit. See FW, Chapter 8.

b.) Derive the energy conservation equation for linear waves on this membrane.

4.) Show that

$$G_i = -\int \sum_k \pi_k \frac{\partial \eta_k}{\partial x_i} dV$$

is a constant of the motion if the Hamiltonian density is not an explicit function of position. The quantity G_i can be identified as the total linear momentum of the field along the x_i direction. The similarity of this theorem with the usual conservation theorem for linear momentum of discrete systems is obvious.

5.) Consider a 1D system, with ϕ fixed at endpoints, such that:

$$\mathcal{L}=\frac{\mu}{2}\left(\partial_t\phi\right)^2-\frac{T}{2}\left(\partial_x\phi\right)^2-m^2\frac{\phi^2}{2}$$

a.) Derive the L EOMs and calculate the dispersion relation of the waves the system supports.

b.) Take the length of the system L very large, so an eikonal approach is valid. Calculate the wave action density two different ways. One of these approaches should start from the Lagrangian density.

c.) Now take $m^2=m^2(x)$ and *slowly* varying in space. How will the wave amplitude change as a packet impinges on a region of increasing $m^2(x)$?

6.) FW 4.13

7.) FW 4.16